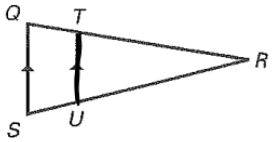
Proportions and Similar Triangles

THEOREM 8.4: TRIANGLE PROPORTIONALITY THEOREM

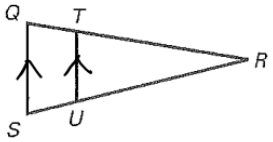
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally



If $\overline{TU} \parallel \overline{QS}$, then $\frac{QT}{TR} = \frac{SU}{UR}$.

THEOREM 8.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

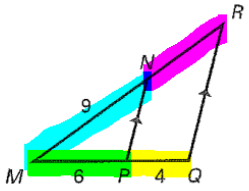
If a line divides two sides of a triangle proportionally, then it is parallel to the 3rd side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$

Example 1 *Finding the Length of a Segment*

What is the length of $\overline{NR}$?



Solution

$\frac{PQ}{MP} = \frac{NR}{MN}$ Theorem ____

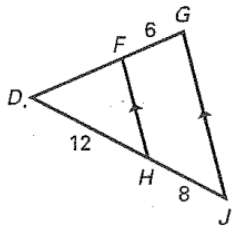
$\frac{4}{6} = \frac{NR}{9}$ Substitute.

$\frac{36}{6} = NR$ Multiply each side by 9.

$6 = NR$ Simplify.

Answer So, the length of $\overline{NR}$ is 6.

1. Find the length of $\overline{DF}$.

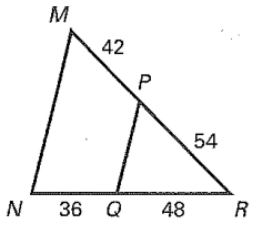


$\frac{DF}{FG} = \frac{DH}{HJ}$

$\frac{x}{6} = \frac{12}{8}$ $x = 9$

$72 = 8x$

2. Given the diagram, determine whether $\overline{MN}$ is parallel to $\overline{PQ}$.



$\frac{36}{48} = \frac{42}{54}$

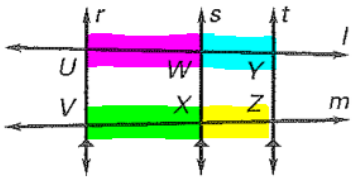
$1944 \neq 2016$

$\frac{3}{4} \neq \frac{7}{9}$

not parallel

THEOREM 8.6

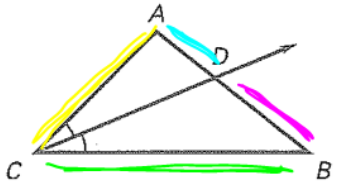
If three parallel lines intersect two transversals, then they divide the transversals proportionally



If $r \parallel s$ and $s \parallel t$, and ℓ and m intersect r, s, and t, then $\frac{UW}{WY} = \frac{VX}{XZ}$.

THEOREM 8.7

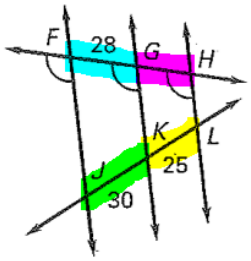
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



If $\overrightarrow{CD}$ bisects $\angle ACB$, then $\frac{AD}{DB} = \frac{AC}{BC}$.

Example 2 *Using Proportionality Theorems*

What is the length of $\overline{GH}$?



Solution

Because corresponding angles are congruent, the lines are parallel and you can use Theorem 8.6.

$\frac{FG}{GH} = \frac{JK}{KL}$ — Parallel lines divide transversals proportionally.

$\frac{28}{GH} = \frac{30}{25}$ — Substitute.

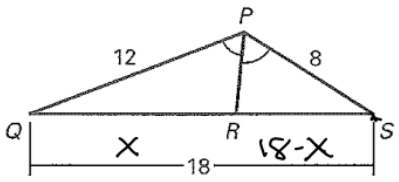
$28 \cdot 25 = 30 \cdot GH$ — Cross product property.

$\frac{700}{30} = GH$ — Divide each side by 30 and simplify.

Answer So, the length of $\overline{GH}$ is $\frac{70}{3}$, or 23.3

Example 3 *Using Proportionality Theorems*

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{QR}$.



Solution

Because $\overline{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 8.7.

Let $x = QR$. Then, $RS = 18 - x$.

$\frac{PS}{QP} = \frac{RS}{QR}$ **Apply Theorem 8.7.**

$\frac{8}{12} = \frac{18-x}{x}$ **Substitute.**

$8 \cdot x = 12(18 - x)$ **Cross product property**

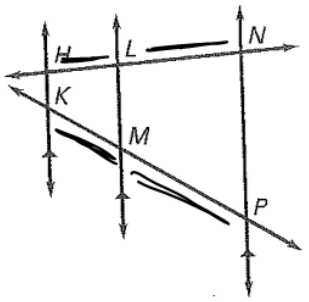
$8x = 216 - 12x$ **Distributive property**

$20x = 216$ **Add $12x$ to each side.**

$x = 10.8$ **Divide each side by 20.**

Answer So, the length of $\overline{QR}$ is 10.8 units.

3. In the diagram, $\overline{HK} \parallel \overline{LM} \parallel \overline{NP}$, $HL = 2.4$, $KM = 2.8$, and $MP = 4.9$. Find the length of $\overline{LN}$.



$\frac{2.4}{x} = \frac{2.8}{4.9}$ $x = 4.2$

$2.8x = 11.76$

4. Find the value of x.

$\frac{16}{12} = \frac{x}{21-x}$ $12x = 336 - 16x$ $x = 12$